Thermoelectric transport through a quantum dot: Effects of asymmetry in Kondo channels

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We consider effects of magnetic field on the thermopower and thermoconductance of a single-electron transistor based on a quantum dot strongly coupled to one of the leads by a single-mode quantum point contact. We show appearance of two new energy scales: $T_{\min}\sim|r|^2E_C(B/B_C)^2$ depending on a ratio of magnetic field B and the field B_C corresponding to a full polarization of point contact and $T_{\max}\sim|r|^2E_C$ depending on a reflection amplitude r and charging energy E_C. We predict that the behavior of thermoelectric coefficients is consistent with the Fermi-liquid theory at temperatures $T\ll T_{\min}$ while crossover from non-Fermi-liquid regime associated with a two-channel Kondo effect to Fermi-liquid single-channel Kondo behavior can be seen at $T_{\min}<T<T_{\max}$.

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The thermoelectric transport through nanostructures is a subject of extensive experimental[1–4] and theoretical[5–8] studies. One of particularly interesting questions is related to the thermoelectric properties of quantum dots (QDs) in a Coulomb blockade (CB) regime. While the experimental behavior of thermoelectric transport through weakly coupled QD devices is well described theoretically,[5,6] the regime of strong coupling of the QD to the reservoirs is far from being understood.

The strong enhancement of thermopower (TP) is important for nanotechnology applications.[4] It provides a challenge for both experimental fabrication of devices with high thermoconductance and theoretical suggestions for efficient mechanisms of heat transfer. On one hand, the nanotechnologies offer fascinating tunability of single electron transport while most parameters can be changed continuously by applying gate voltage, external electric and magnetic fields, etc. On the other hand, the nanodevices efficiently operate in strong-coupling regime where effects of strong electron correlations can be viewed as a prominent mechanism for the thermoelectric coefficients enhancement. In particular, the Kondo effect is known as a tool for strong intensification of electric transport through single-electron transistor (SET). Moreover, by increasing the number of channels one can fine tune SET to a non-Fermi-liquid (NFL) regime. The NFL behavior is however illusive being very sensitive to variation in external parameters since channel symmetry is generally unprotected.[9] A special case is a situation where the symmetry of channels follows from one of the basic symmetry, e.g., time-reversal symmetry[6] and thus the NFL behavior is especially robust. In this case magnetic field is a relevant perturbation which drives the system to a different universality class characterized by a restoration of the FL behavior. Such effects have been discussed in literature[10] in connection with the *thermodynamics* (capacitance and charge fluctuations) of a quantum dot. In this Brief Report we present a theory of an interplay between NFL and FL strong-coupling regimes in *thermoelectric transport* through the nanostructures controlled by magnetic field.

Typical experimental setup[3] for measuring the thermopower $S=-\Delta V_{th}/\Delta T$ is shown on Fig. 1(a). The measurement of the thermovoltage ΔV_{th} provides independent information on the thermoconductance G_T. The temperature difference across the dot ΔT is controlled by using a current heating technique. The differential conductance G is measured at variable gate voltages V_g. Similar to differential conductance, the thermopower $S=G_T/G$ shows the oscillations as a function of V_g. However, these oscillations are not sinusoidal at low temperatures. Moreover, no relation analogous to Cutler-Mott formula[11] $S\sim\partial\ln G/\partial V_g$ exists in that limit underlining importance of strong electron correlations.

Theory of TP of a CB quantum dot has been studied[5] in the framework of sequential tunneling and verified by experiment.[1,2] This theory gives qualitatively correct results at high temperatures for a weak coupling of the dot to reservoirs. However, at low temperatures in the Coulomb blockade valleys, the main mechanism of thermotransport is the interaction-induced cotunneling[7,12] ignored in Ref. 5. By increasing the coupling of the SET to one (or both) reservoirs one reaches the strong-coupling regime where Kondo physics becomes important.[13–15]

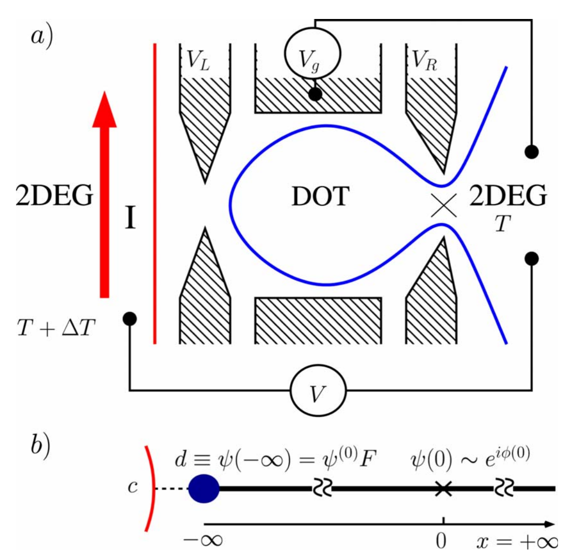

FIG. 1. (Color online) Top: (a) experimental setup of SET in the strong-coupling regime (see text for the details). The arrow along left lead stands for the electric current controlling the Joule heat. Almost transparent QPC remaining at the reference temperature T is denoted by the cross. Bottom: (b) equivalent circuit described by Hamiltonians (4)–(7).

The theory of the TP of the SET designed as a quantum dot strongly coupled to one of the leads through almost transparent quantum point contact (QPC) (Ref. 16) and weakly coupled to a second lead has been developed by Andreev and Matveev (AM) in Ref. 6. It is based on the Kondo physics. However, the role of the impurity spin in the ordinary Kondo problem is played by the orbital degrees of freedom: the left/right movers at QPC stand for the impurity spin-up/down projections in the conventional Kondo problem while two spin projections of an electron account for the scattering channels.[17] The impurity spin-flip process in the ordinary Kondo effect is represented in this model by the reflection in the QPC, with the Kondo coupling determined by the reflection coefficient. The two limits were considered in Ref. 6: (i) the electron spins are fully polarized by strong external magnetic field B (Ref. 18) and (ii) the electrons spins are unpolarized, $B=0$. In the first limit, corresponding to the single channel Kondo (CK) physics, the TP shows sinusoidal oscillations as a function of the gate voltage V_g having nodes both in Coulomb valleys (N is integer) and peaks (N is half integer)

$$S \sim \frac{1}{e}|r|\frac{T}{E_C}\sin[2\pi N(V_g)]. \tag{1}$$

Here $N(V_g)$ is a dimensionless parameter which is proportional to V_g. The TP is a linear function of both reflection amplitude $|r|$ and temperature. We refer to the linear-T dependence of TP as the FL regime. However, in contrast to TP in bulk metals $S \sim T/\epsilon_F$, the charging energy E_C in the denominator of Eq. (1) is much smaller[19] than the Fermi energy ϵ_F (cf. Ref. 20) thus making the TP strongly enhanced.

There are two important regimes for thermoelectric coefficients in the limit of unpolarized electrons. First, at temperatures $E_C|r|^2 \ll T \ll E_C$ the TP is sinusoidal in $N(V_g)$ and quadratic in the reflection amplitude $|r|$,

$$S \sim \frac{1}{e}|r|^2 \ln\left[\frac{E_C}{T}\right]\sin[2\pi N(V_g)]. \tag{2}$$

Second, at smaller temperatures $T \ll E_C|r|^2$, the TP oscillations are nonsinusoidal

$$S \sim \frac{1}{e}|r|^2\frac{T}{\Gamma}\ln\left[\frac{E_C}{T+\Gamma}\right]\sin[2\pi N(V_g)]f\left(\frac{T}{\Gamma}\right), \tag{3}$$

where $\Gamma=\Gamma(V_g) \sim E_C|r|^2\cos^2[\pi N(V_g)]$ and $f(x)$ is defined in Ref. 21. The maximum value of S scales as $S_{\max} \sim e^{-1}|r|\sqrt{T/E_C}\,\ln(E_C/T)$. Thus, the TP is strongly enhanced by the electron's correlations. We refer to $\sqrt{T}\ln T$ behavior of $S_{\max}$ as the NFL regime. Such scaling of the TP at $T \ll E_C$ is attributed to the two-CK (2CK) effect.[22,23] However, as it is known,[22] the 2CK strong-coupling fixed point is unstable if not protected by the basic symmetry. This leads, in particular, to smearing of Coulomb staircase steps due to magnetic field driven asymmetry of reflection amplitudes.[10]

In this Brief Report we show that the effect of magnetic field on thermopower is much more pronounced: instead of a divergent at zero temperature $S_{\max}/T \sim 1/\sqrt{T}$ for $B=0$, at a finite B there is a region of low temperatures $T < T_{\min} \sim B^2$, where $S_{\max}$ saturates (see Fig. 2, right panel). Below we present an explicit expression for this crossover for different parameters of the problem.

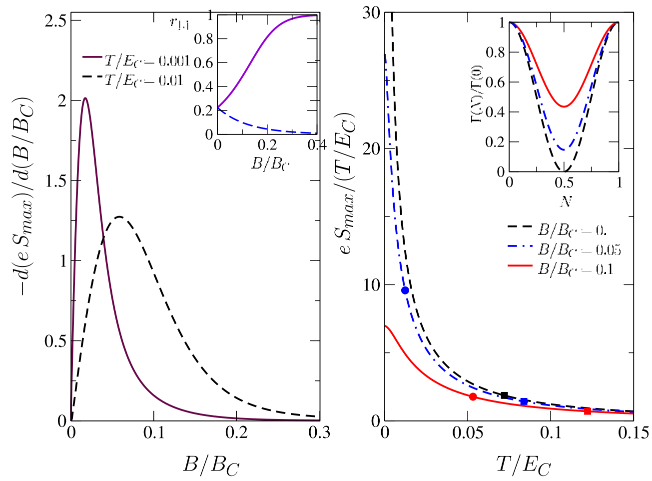

FIG. 2. (Color online) Left panel: $-d(eS_{\max})/dx$ as a function of $x=B/B_C$ for $T \ll T_{\max}=\Gamma(0)$ (solid line) and $T \approx T_{\max}$ (dashed line). Maximum corresponds to $T_{\min}(B)=\Gamma(\frac{1}{2}) \sim T$. Inset: $|r_\uparrow|$ and $|r_\downarrow|$ as function of B/B_C. Right panel: $eS_{\max}/(T/E_C)$ as a function of T/E_C. Circles and squares indicate positions of $T_{\min}$ and $T_{\max}$ correspondingly. Inset: Γ as a function of V_g at zero and finite B's. Lines correspond to the same set of parameters as on the main frame, $|r_0|^2=0.05$ for all curves.

The Hamiltonian describing the quantum dot coupled weakly to the left contact and strongly to the right contact [Fig. 1(b)] has the form $H=H_0+H_L+H_R+H_C$, where

$$H_0 = \sum_{k,\alpha}\epsilon_{k,\alpha}c^\dagger_{k,\alpha}c_{k,\alpha} + \sum_\alpha \epsilon_\alpha d^\dagger_\alpha d_\alpha + \sum_\alpha \frac{v_{F,\alpha}}{2\pi}\int_{-\infty}^{\infty}\{[\Pi_\alpha(x)]^2 + [\partial_x\phi_\alpha(x)]^2\}dx \tag{4}$$

describes a noninteracting part, c denotes the electrons in the left lead, and d stands for the electrons in the dot. Here $\alpha=\uparrow,\downarrow$, ϕ_α is a bosonization displacement operator describing transport through the QPC with a scatterer at $x=0$, and Π_α is the conjugated momentum $[\phi_\alpha(x),\Pi_{\alpha'}(x')]=i\pi\delta(x-x')\delta_{\alpha\alpha'}$. Note that the operator $d_\alpha=\psi_\alpha(-\infty)$ can be expressed through the fermionic operator $\psi_\alpha(x) \sim e^{i\phi_\alpha(x)}$ in the one-dimensional channel describing the QPC [Fig. 1(b)].

The Hamiltonian H_L describes the tunneling from the left (hot) lead to the dot ($t_{k,\alpha}$ is a tunnel amplitude)

$$H_L = \sum_{k,\alpha}(t_{k,\alpha}c^\dagger_{k,\alpha}d_\alpha + \text{H.c.}). \tag{5}$$

The Hamiltonian H_R accounts for the backward scattering in the QPC, r_α, are reflection amplitudes for $\uparrow,\downarrow$

$$H_R = -\frac{D}{\pi}\sum_\alpha |r_\alpha|\cos[2\phi_\alpha(0)], \tag{6}$$

D is a bandwidth.

The Hamiltonian H_C describes the Coulomb interaction in the dot

$$H_C = E_C\left[\hat{n} + \frac{1}{\pi}\sum_{\alpha}\phi_{\alpha}(0) - N(V_g)\right]^2. \quad (7)$$

Here the fluctuating charge (in units of e) in the dot area [corresponding to $x<0$ in the equivalent circuit in Fig. 1(b)] is equal[24] to an integer $\hat{n}+N$ minus $\int_0^\infty \Sigma_\alpha \psi_\alpha^\dagger(x)\psi_\alpha(x)dx = \pi^{-1}\Sigma_\alpha\phi_\alpha(0)$, where $\hat{n}$ takes values 0, 1 and the c-number N is absorbed in $N(V_g)$.

The electric current and thermocurrent through the dot are expressed in this approximation in terms of the Matsubara's Green's function (GF)

$$\mathcal{G}(\tau) = -\sum_{\alpha}\langle T_\tau d_\alpha(\tau)d_\alpha^\dagger(0)\rangle$$

$$= -\sum_{\alpha}\langle T_\tau \psi_\alpha^{(0)}(\tau)\hat{F}(\tau)\hat{F}^\dagger(0)\psi_\alpha^{(0)\dagger}(0)\rangle, \quad (8)$$

where $\psi_\alpha^{(0)}(x,\tau)=e^{\mp ik_{F,\alpha}x}\psi_\alpha^{(0)}(\tau)$ are the Fermi operators of the free left (right) movers in the one-dimensional channel describing the QPC. We denote by definition [Fig. 1(b)] $\psi_\alpha^{(0)}(\tau)\hat{F}(\tau)=\hat{d}_\alpha(\tau)$. The operator $\hat{F}$ obeys the commutation relation $[\hat{F},\hat{n}]=\hat{F}$ (see Ref. 25) and takes into account effects of interaction and reflection given by Eqs. (6) and (7).

Since the operators $\psi_\alpha^{(0)}$ and $\hat{F}$ are decoupled, the GF is factorized into $\mathcal{G}(\tau)=G_0(\tau)K(\tau)$, with $G_0(\tau)=-\nu_0\pi T/\sin(\pi T\tau)$ being the free electrons GF, ν_0 is the density of states in the dot without interaction, and $K(\tau)=\langle T_\tau\hat{F}(\tau)\hat{F}^\dagger(0)\rangle$ accounts for interaction effects. The electric conductance[14] is given by

$$G = \frac{G_L\pi T}{2}\int_{-\infty}^{\infty}\frac{1}{\cosh^2(\pi Tt)}K\left(\frac{1}{2T}+it\right)dt. \quad (9)$$

The thermoconductance takes the form[6]

$$G_T = -\frac{i\pi^2}{2}\frac{G_L T}{e}\int_{-\infty}^{\infty}\frac{\sinh(\pi Tt)}{\cosh^3(\pi Tt)}K\left(\frac{1}{2T}+it\right)dt. \quad (10)$$

Here $G_L \ll e^2/h$ denotes the tunnel conductance of the left barrier calculated ignoring influence of the dot.

In order to calculate the thermoelectric coefficients for the models [Eqs. (4)–(7)] we generalize AM theory for the case of finite magnetic field. The detailed calculations will be published elsewhere.[26] New effects reported in the present Brief Report and missing in AM theory appear due to asymmetry of the point-contact reflection amplitudes (cf. Ref. 10). This asymmetry, in turn, leads to the asymmetry of the channels in 2CK. Besides, the magnetic field lifts out the spin-charge separation characteristic for the unpolarized models.[27] However, the spin-charge separation survives if the effect of magnetic field reduces only to changing the reflection amplitudes $r_\uparrow$ and $r_\downarrow$ due to different Fermi momenta of $\uparrow$ and $\downarrow$ electrons caused by Zeeman splitting. As the reflection coefficients determine the coupling constants in the equivalent Kondo problem the effect of magnetic field induced asymmetry of $|r|$'s is likely to be the major effect of magnetic field at weak scattering. Moreover, we will show below (see Fig. 2) that the significant effect of magnetic field at low temperatures occurs at $B \ll B_C$, where B_C is the field corresponding to full polarization of the QPC, which makes the principle results of this study model independent.

In the spirit of AM theory[6] we first calculate the leading in the reflection amplitudes r_α corrections to the thermoconductance G_T and the thermopower S. Thus, all perturbative corrections are powers of a symmetric $s=|r_\uparrow|+|r_\downarrow|\approx 2|r_0|$ and antisymmetric $a=||r_\downarrow|-|r_\uparrow||\sim|r_0 B/B_C|$ combinations of the reflection amplitudes. Here $|r_0|$ stands for the reflection amplitude at $B=0$. In the leading order we reproduce Eq. (2) with $|r_0|^2$ replaced by $|r_\uparrow r_\downarrow|$. At large enough magnetic fields $B>B^*$ when the channel $\downarrow$ becomes almost completely reflecting and $|r_\downarrow|\approx 1$, one obtains $|r_\uparrow r_\downarrow|\approx|r_\uparrow|$ which explains the crossover from $\sim|r|^2$ for small B [Eq. (2)] to $\sim|r|$ for $B>B^*$ [Eq. (1)] in TP. Proceeding with the higher order $\sim|r|^4$ perturbative corrections to thermoconductance and TP we notice existence of a new energy scale Γ see Eq. (11) depending on external magnetic field.[26]

More efficient way to prove emergence of new energy scale is to map the model [Eqs. (4)–(7)] onto effective Anderson model.[13] This mapping, being nonperturbative in s and a accounts for low-frequency dynamics of the spin modes. The channel asymmetry a leads to a nontrivial contribution to the Kondo-resonance width Γ in the vicinity of Coulomb peaks

$$\Gamma(N) = \frac{2\gamma E_C}{\pi^2}[s^2\cos^2(\pi N) + a^2\sin^2(\pi N)]. \quad (11)$$

As it is shown in the right inset of Fig. 2, at $B\neq 0$ the width Γ acquires a gap $T_{\min}=\Gamma(\frac{1}{2})\sim a^2E_C\sim E_C(B/B_C)^2\ll T_{\max}=\Gamma(0)$. The thermal and electric conductances take the form

$$G_T = -\frac{1}{12\pi}\frac{G_L T^3}{eE_C\Gamma^2}(s^2-a^2)\ln\left[\frac{E_C}{T+\Gamma}\right]\sin(2\pi N)F_1\left(\frac{T}{\Gamma}\right),$$

$$G = \frac{G_L T^2}{4\gamma E_C\Gamma}F_2\left(\frac{T}{\Gamma}\right). \quad (12)$$

The functions $F_1(x)$ and $F_2(x)$ are defined in Ref. 21, $\gamma\approx 1.78$. Equation (12) allow regular expansion at low temperatures $T\ll T_{\min}$, where $F_1(x\ll 1)=8\pi^4/5-(136\pi^6/35)x^2+O(x^4)$ and $F_2(x\ll 1)=8\pi^2/3-(8\pi^4/5)x^2+O(x^4)$. Taking the leading term of this expansion we obtain

$$S = -\frac{\pi\gamma}{5e}\frac{T}{\Gamma(N)}(s^2-a^2)\ln\left[\frac{E_C}{\Gamma(N)}\right]\sin(2\pi N), \quad (13)$$

where $s,\ a=||r_\downarrow|\pm|r_\uparrow||$ and $\Gamma(N)$ given by Eq. (11). Equations (11)–(13) and emergence of new energy scales $T_{\min}$ and $T_{\max}$ represent the central result of this Letter. One sees from Eq. (13) that the thermopower at $a\ll s$ is not sinusoidal. Moreover, the extrema of S at $T\ll T_{\min}$ are located at gate voltages $N\approx 1/2\pm\pi a/2s$ and its value at the maximum is

$$S_{\max}\sim\frac{1}{e}\frac{T}{E_C}\frac{s}{a}\ln\left(\frac{1}{a}\right). \quad (14)$$

Thus, the Fermi-liquid behavior $eS_{\max}\sim T/E_0$ is nontrivially restored. The role of ϵ_F in Ref. 11 is played by $E_0=E_C(B/B_C)\ln^{-1}(B_C/B|r|)$ (Ref. 19) which vanishes at

$B=0$ (giant Fermi liquid) and has very weak (logarithmic) dependence on the reflection amplitude.

Existence of the new energy scale $T_{\min}$ manifests itself in the behavior of $S_{\max}(T)/T$ (see Fig. 2, right panel), diverging at $B=0$ while saturating at temperatures below $T_{\min}$ at a nonzero B. Another manifestation of the new energy scale is the existence of maximum in dS/dB (Fig. 2, left panel). This maximum is located at magnetic field B_m where $T_{\min}(B_m)\sim T$.

Finally we identify three regions of parameters with different behavior of the thermopower. (i) "Giant FL" regime Eq. (13) is predicted for $T<T_{\min}$. (ii) Proximity to "strong NFL" regime with $S_{\max}\sim e^{-1}|r|\sqrt{T/E_C}\ln(E_C/T)$ can be seen at $T_{\min}<T<T_{\max}$. (iii) Perturbative "weak NFL" regime Eq. (2) holds at $T_{\max}<T<E_C$. Thus, the magnetic field stabilizes the FL thermoelectric properties and can be used as a fine tuning parameter to control the heat flow.

We conclude that the external magnetic field applied to the SET in the strong-coupling regime is responsible for the NFL-to-FL crossover in the transport properties. A parallel magnetic field applied to SET results in the channel up/down asymmetry and thus changes the universality class from the two-channel Kondo to the single-channel Kondo regimes. We also expect restoration of the FL transport for the out-of-equilibrium SETs, in particular, for SETs at a finite source-drain voltage and subject to external noise.

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